Statics

Chapter five

Centrifugal pumps

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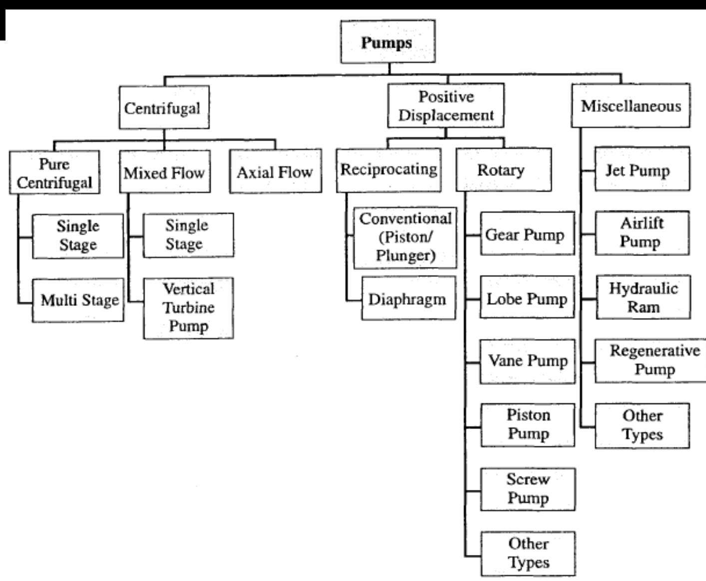
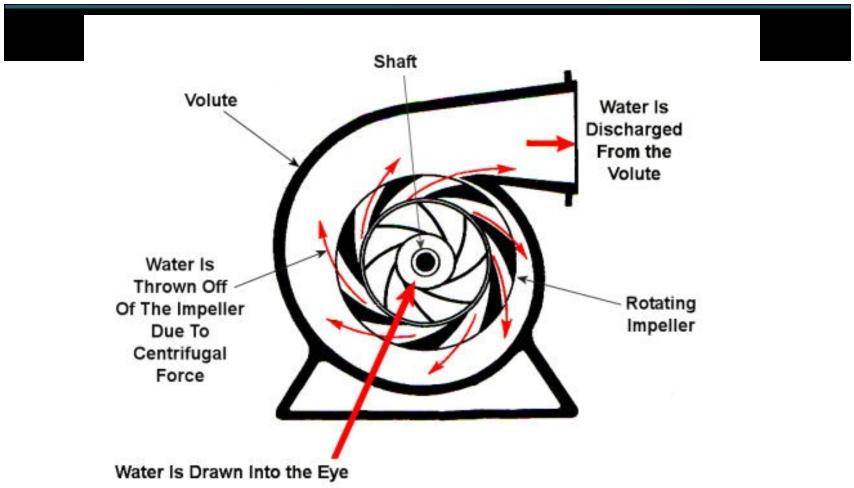
Centrifugal pumps



Centrifugal pumps



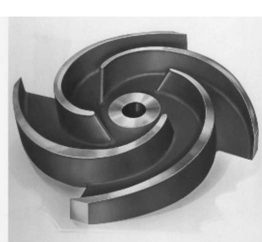
Centrifugal pumps

5.2 COMPONENT PARTS OF A CENTRIFUGAL PUMP

5.2.1 Impeller

1. *Shrouded/Fully Enclosed Impellers*
2. *Semi-enclosed Impellers*
3. *Fully Open Impeller*


Shrouded impeller


Semi-enclosed impeller


Fully open impeller

Centrifugal pumps

5.2.2 Casing

1. ***Volute Casing***
2. ***Double-volute Casing***
3. ***Turbine Pump (Casing with Guide Vanes)***

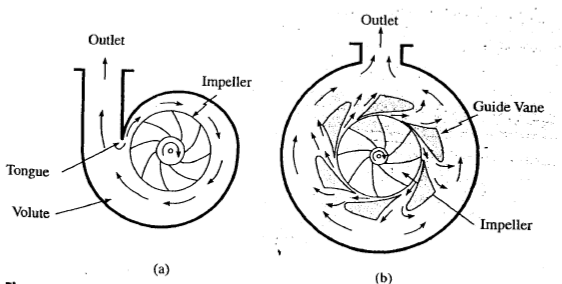


Fig. 5.4 *Types of centrifugal pumps: (a) Volute pump (b) Turbine pump*

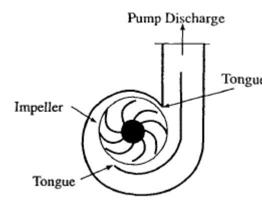


Fig. 5.5 *Double-volute casing (schematic)*

5.2.3 Suction Pipe with Strainer and Foot Valve

5.2.4 Delivery Pipe with Control Valve

Centrifugal pumps

5.2.5 Classifications

Table 5.1 *Classification of centrifugal pumps*

Basis of Classification	Types
Type of casing	• Volute (single volute, double volute) • Turbine type
Number of stages	• Single • Multistage
Type of suction inlet	• Single suction • Double suction
Impeller types	• Closed • Semi-Open • Open
Construction of casing	• Vertical split • Horizontal split
Axis of rotation	• Horizontal • Vertical • Inclined
Basis of flow direction	• Radial flow • Mixed flow • Axial flow

Centrifugal pumps

5.3 EULER'S EQUATION FOR CENTRIFUGAL PUMP

If $\hat{r}$ is the position vector in a curvilinear motion of a fluid, $\hat{F}$ is the external force vector and $\hat{M}$ is the linear momentum vector, the moment-of-momentum principle states that

$$\left(\hat{r}\times\hat{F}\right)=\frac{d}{dt}\left(\hat{r}\times\hat{M}\right) \tag{5.1}$$

If the moment of external forces ($\hat{r}\times\hat{F}$) is replaced by torque $\hat{T}$ then

$$\hat{T}=\frac{d}{dt}(\hat{r}\times\hat{M}) \tag{5.2}$$

Centrifugal pumps

5.3.2 Euler Equation for Centrifugal Pump

Let

r_1 and r_2 = Radii of fluid element at entrance and exit

v_{r1} and v_{r2} = Relative velocities at entrance and exit

V_1 and V_2 = Absolute velocities at entrance and exit

ω = Angular velocity of the impeller

N = Revolutions per minute of the impeller

Note that the angular velocity of the impeller $\omega=\dfrac{2\pi N}{60}$

u = Tangential velocity of the blade at any radius $r=\omega r$

$u_1=\omega r_1=\dfrac{\pi D_1 N}{60}$ where D_1 = Outer diameter of the impeller

$u_2=\omega r_2=\dfrac{\pi D_2 N}{60}$ where D_2 = Inner diameter of the impeller

Centrifugal pumps

For a steady, frictionless system,

Torque exerted by the fluid on the rotor = Decrease in the rate of change of moment of momentum

= [Rate of change of moment of momentum of fluid going **in to** the control volume] – [rate of change of moment of momentum of fluid going **out of** the control volume]

Torque $T =$ [Rate of mass flow through the impeller] × [Increase of moment of momentum of the fluid in each blade]

$$= \dot{m} \times (r_2 V_2 \cos \alpha_2 - r_1 V_1 \cos \alpha_1) \quad (5.3)$$

Since $\rho =$ Density of water, is constant,

$\dot{m} =$ Rate of mass flow through the impeller $= \rho Q$

where $Q =$ Total discharge entering the impeller.

Now $T = \rho Q (r_2 V_2 \cos \alpha_2 - r_1 V_1 \cos \alpha_1)$ (5.4)

Energy transferred in unit time = Power transmitted by the impeller to water

$$P = T\omega = \rho Q \omega (r_2 V_2 \cos \alpha_2 - r_1 V_1 \cos \alpha_1) \quad (5.5)$$

Since $\omega r = u =$ Tangential component of the impeller at radius r,

$u_1 = \omega r_1$ and $u_2 = \omega r_2$

$$P = \rho Q (u_2 V_2 \cos \alpha_2 - u_1 V_1 \cos \alpha_1) \quad (5.6)$$

This equation (5.6) is known as *Euler's equation relating power in a centrifugal pump.*

Centrifugal pumps

5.3.3 Alternate Forms of Euler Equation

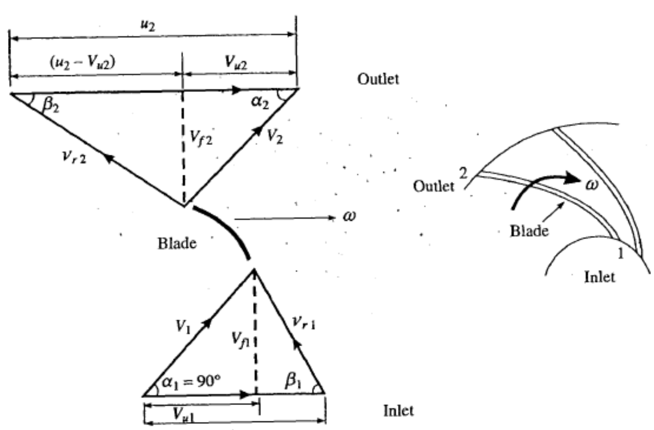


Fig. 5.6 *Generalized velocity triangles at inlet and outlet of the impeller of a pump*

Centrifugal pumps

$V_1 \sin \alpha_1 = V_{f1}$ = Flow component of absolute velocity V_1
$V_2 \cos \alpha_2 = V_{f2}$ = Flow component of absolute velocity V_2
N = Rotational speed of the impeller in rpm

Referring to Fig. 5.6, from the velocity triangle at outlet,

$$V_{f2}^2 = v_{r2}^2 - (u_2 - V_{u2})^2 \tag{5.7}$$

$$V_2^2 = V_{u2}^2 + V_{f2}^2 = V_{u2}^2 + v_{r2}^2 - (u_2^2 - V_{u2}^2) \tag{5.8}$$

$$= V_{u2}^2 + v_{r2}^2 - u_2^2 - V_{u2}^2 + 2u_2V_{u2}$$

$$2u_2V_{u2} = V_2^2 - v_{r2}^2 + u_2^2$$

$$u_2V_{u2} = (V_2^2 - v_{r2}^2 + u_2^2)/2 \tag{5.9}$$

Similarly, from the velocity triangle at the inlet,

$$V_1^2 = V_{u1}^2 + V_{f1}^2 = V_{u1}^2 + v_{r1}^2 - (u_1^2 - V_{u1}^2)$$

$$V_1^2 = V_{r1}^2 - u_1^2 + 2u_1V_{u1}$$

$$u_1V_{u1} = (V_1^2 - v_{r1}^2 + u_1^2)/2 \tag{5.9-a}$$

Thus, from Eq. 5.6 the power transferred = energy transmitted by the impeller to water per unit time is

$$P = \rho Q\,(u_2V_2 \cos \alpha_2 - u_1V_1 \cos \alpha_1) = \rho Q\,(u_2V_{u2} - u_1V_{u1}) \tag{5.10}$$

Equation 5.10 is a very commonly used form of *Euler equation for power* in a pump.

Substituting the results of equations (5.9 and 5.9-a) in Eq. (5.10),

$$P = \frac{\rho Q}{2}\left(V_2^2 - v_{r2}^2 + u_2^2 - V_1^2 + v_{r1}^2 - u_1^2\right)$$

$$= \rho Q\left[\frac{(V_2^2 - V_1^2)}{2} + \frac{(v_{r1}^2 - v_{r2}^2)}{2} + \frac{(u_2^2 - u_1^2)}{2}\right] \tag{5.11}$$

Centrifugal pumps

Considering the energy head H_e = energy per unit weight of fluid transferred to the fluid by the impeller,

$$H_e = \frac{\rho Q}{\rho Q g}\left[\frac{(V_2^2 - V_1^2)}{2} + \frac{(v_{r1}^2 - v_{r2}^2)}{2} + \frac{(u_2^2 - u_1^2)}{2}\right]$$

$$H_e = \left[\frac{(V_2^2 - V_1^2)}{2g} + \frac{(v_{r1}^2 - v_{r2}^2)}{2g} + \frac{(u_2^2 - u_1^2)}{2g}\right] \tag{5.12}$$

Also by Eq. (5.9) $H_e = \frac{1}{g}(u_2V_{u2} - u_1V_{u1})$ (5.13)

Equation 5.13 is the *Euler equation for head* in a centrifugal pump.

H_e is called *Euler head* and represents the head transferred to water by the impeller. In this,

- The first term $\frac{(V_2^2 - V_1^2)}{2g}$ represents the increase in kinetic energy.
- The second term $\frac{(u_2^2 - u_1^2)}{2g}$ represents the increase in static pressure due to centrifugal action.
- The third term $\frac{(v_{r1}^2 - v_{r2}^2)}{2g}$ indicates the change in the kinetic energy due to retardation of flow.

Centrifugal pumps

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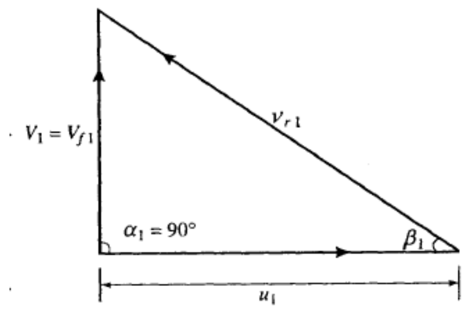
From Eq. (5.13), the Euler head, for practical use, is thus

$$H_e = \frac{1}{g}(u_2 V_{u2}) \qquad (5.14)$$



Radial Entry

Centrifugal pumps

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5.4 ANALYSIS BASED ON EULER'S EQUATION

1. Static Head

$$H_{stat} = h_s + h_d$$

static suction lift h_s

static delivery lift h_d

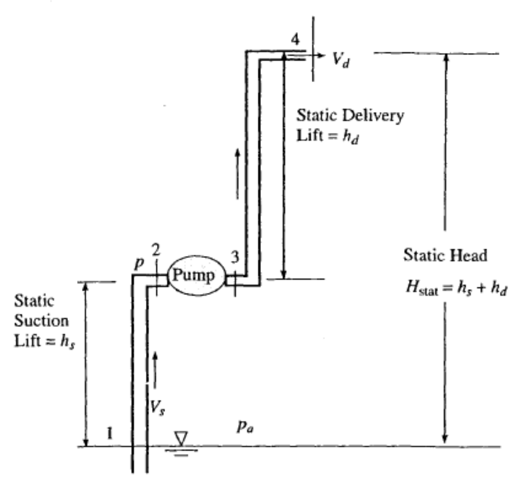

Fig. 5.8 *Definition sketch of a pump set-up*

Centrifugal pumps

2. Euler Head (Theoretical Head) of a Pump

$$H_e = \frac{V_{u2}u_2}{g}$$

3. Manometric Head

$$H_m = H_e - h_{fi}$$

Hydraulic losses

manometric efficiency

$$\eta_{ma} = \frac{H_m}{H_e} = \frac{gH_m}{V_{u2}u_2}$$

Centrifugal pumps

Expressions for Manometric Head

Apply Bernoulli's theorem to Section 1 and Section 4 by taking the liquid level in the sump as datum.

$$\left(\frac{p_a}{\gamma} + 0 + 0\right) + H_m = \left(\frac{p_a}{\gamma} + (h_s + h_d) + (h_{fs} + h_{fd}) + \frac{V_d^2}{2g}\right) \qquad (5.16)$$

where h_{fs} = Frictional losses including minor losses in the suction pipe

h_{fd} = Frictional losses including minor losses in the delivery pipe

V_d = Discharge velocity at the delivery pipe

Hence, $H_m = (h_s + h_d) + (h_{fs} + h_{fd}) + \frac{V_d^2}{2g} = H_{stat} + \frac{V_d^2}{2g} + \Sigma$ Losses (5.17)

In most applications, $\frac{V_d^2}{2g}$ is usually neglected as too small or is included in the minor losses. The manometer head H_m is then taken as

$$H_m = (h_s + h_d) + (h_{fs} + h_{fd}) = H_{stat} + \Sigma \text{ Losses} \qquad (5.17\text{-a})$$

Centrifugal pumps

[redacted]

The difference of energy heads between the outlet flange and the inlet flange of the pump should also be equal to the manometer head, H_m. Thus, considering the sections 2 and 3 to be at the same elevation,

$$\left(\frac{p_3}{\gamma}+\frac{V_d^2}{2g}\right)-\left(\frac{p_2}{\gamma}+\frac{V_s^2}{2g}\right)=H_m$$

$$\left(\frac{p_3}{\gamma}-\frac{p_2}{\gamma}\right)=H_m-\left(\frac{V_d^2}{2g}-\frac{V_s^2}{2g}\right)$$

If $V_d = V_s$ or if the difference between the two velocity heads is negligibly small then

$$\left(\frac{p_3}{\gamma}-\frac{p_2}{\gamma}\right)=H_m \qquad (5.17\text{-b})$$

Centrifugal pumps

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5.4.2 Ideal Increase in Pressure Head in the Impeller

Energy of liquid at inlet + Energy added externally by the pump to the liquid = Energy of liquid at outlet

$$\left(\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}\right)+\frac{u_2V_{u2}}{g}=\left(\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}\right)$$

Increase in piezometric head at the pump =

$$\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)=\Delta H_p=\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}+\frac{u_2V_{u2}}{g}\right) \qquad (5.18)$$

Thus, for $Z_1 = Z_2$, for the ideal case of no losses, the relationship between H_m and H_e can be expressed as

$$\left(\frac{p_2}{\gamma}-\frac{p_1}{\gamma}\right)=H_m=\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)+\frac{u_2V_{u2}}{g}$$

ΔK.E

Centrifugal pumps

1. Expression for ΔH_p under Ideal Conditions

$$\frac{V_{f2}}{(u_2 - V_{u2})} = \tan\beta_2$$

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan\beta_2} = u_2 - V_{f2}\cot\beta_2$$

$$V_2^2 = V_{u2}^2 + V_{f2}^2$$
$$= (u_2^2 + V_{f2}^2\cot^2\beta_2 - 2u_2 V_{f2}\cot\beta_2 + V_{f2}^2)$$
$$= V_{f2}^2(1 + \cot^2\beta_2) + u_2^2 - 2u_2V_{f2}\cot\beta_2$$
$$= V_{f2}^2\operatorname{cosec}^2\beta_2 + u_2^2 - 2u_2V_{f2}\cot\beta_2$$

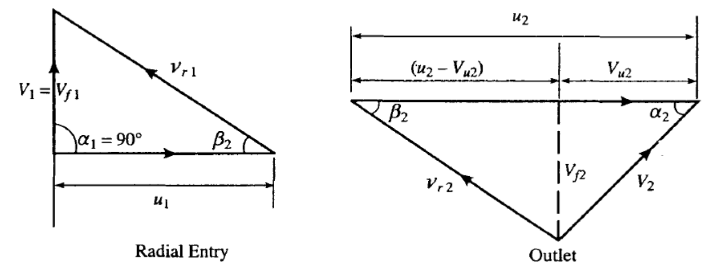

Fig. 5.9 *Radial entry and outlet velocity triangle of a pump*

Centrifugal pumps

$$\Delta H_p = \frac{\Delta p_i}{\gamma} = \frac{1}{2g}\left[V_1^2 - V_2^2 + 2V_{u2}u_2\right]$$

$$\frac{\Delta p_i}{\gamma} = \frac{1}{2g}\left[V_{f1}^2 - V_{f2}^2\operatorname{cosec}^2\beta_2 - u_2^2 + 2u_2V_{f2}\cot\beta_2 + 2(u_2 - V_{f2}\cot\beta_2)u_2\right]$$

$$\frac{\Delta p_i}{\gamma} = \frac{1}{2g}\left[V_{f1}^2 - V_{f2}^2\operatorname{cosec}^2\beta_2 - u_2^2\right] \qquad (5.20)$$

Centrifugal pumps

*EXAMPLE 5.1

A centrifugal pump has an impeller of 30 cm outer diameter. The vane tips are radial at the outlet. For a rotative speed of 1450 rpm, calculate the manometric head developed. Assume a manometric efficiency of 82%.

Solution

Given: D_2 = 0.30 m, N = 1450 rpm, $\eta_o = 0.82$

Consider the outlet velocity triangle shown in Fig. 5.18. From this,

$V_2 \cos \alpha_2 = V_{u2} = u_2$

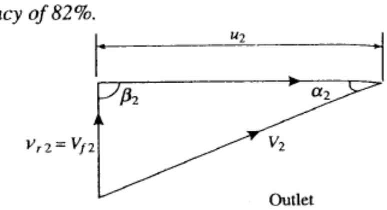

Fig. 5.18 *Outlet velocity triangle, Example 5.1*

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1450}{60} = 22.78 \text{ m/s}$$

$$\text{Manometric efficiency} = \eta_{ma} = \frac{gH_m}{u_2 V_{u2}} = \frac{gH_m}{u_2^2}$$

$$0.82 = \frac{9.81 \times H_m}{(22.78)^2}$$

H_m = Manometric head developed = 43.38 m

Centrifugal pumps

*EXAMPLE 5.2

A centrifugal pump delivers water against a total head of 10 m at a design speed of 1000 rpm. The vanes are curved backwards and make an angle of 30° with the tangent at the outer periphery of the impeller. The impeller diameter is 30 cm and has a width of 5 cm at the outlet. (a) If the manometric efficiency is 0.95%, estimate the discharge of the pump. (b) Assuming an overall efficiency of 76%, estimate the power required to drive the pump.

Solution

Given: H_m = 10.0 m, N = 1000 rpm, β_2 = 30°, D_2 = 0.30 m, b_2 = 0.05 m, η_{ma} = 0.95, η_0 = 0.76

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.708 \text{ m/s}$$

(a) Manometric efficiency $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}}$

$$0.95 = \frac{9.81 \times 10.0}{15.708 \times V_{u2}}$$

V_{u2} = 6.574 m/s

From the outlet velocity triangle, since $\beta_2 < 90°$

$$\tan \beta_2 = \frac{V_{f2}}{(u_2 - V_{u2})}$$

Centrifugal pumps

$$\tan 30° = \frac{V_{f2}}{(15.708 - 6.574)}$$

$$V_{f2} = 5.274 \text{ m/s}$$

$$\text{Discharge} = Q = \pi D_2 b_2 V_{f2}$$
$$= \pi \times 0.30 \times 0.05 \times 5.274$$
$$Q = 0.249 \text{ m}^3/\text{s} = 249 \text{ L/s}$$

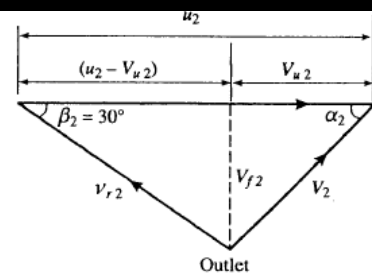

Fig. 5.19 *Outlet velocity triangle, Example 5.2*

(b) Power required to drive the pump

$$= P_s = \frac{\gamma Q H_m}{\eta_0} = \frac{9.79 \times 0.249 \times 10}{0.76} = 32.0 \text{ kW}$$

Centrifugal pumps

*****EXAMPLE 5.7**

A centrifugal pump lifts water from a sump to an overhead reservoir. The static lift is 40 m out of which 3 m is the suction lift. The suction and delivery pipes are both of 35 cm diameter. The friction loss in suction pipe is 2.0 m and in delivery pipe it is 6.0 m. The impeller is 0.5 m in diameter and has a width of 3 cm at the outlet. The speed of the pump is 1200 rpm. The exit blade angle is 20°. If the manometric efficiency is 85%, determine the pressures at the suction and delivery ends of the pump and the discharge. Assume that the inlet and outlet of the pump are at the same elevation.

Solution

$D_2 = 0.50$ m, $b_2 = 0.03$ m, $h_s = 3.0$ m, $h_d = 37$ m, $h_{fs} = 2.0$ m, $h_{fd} = 6$ m, $N = 1200$ rpm, $\beta_2 = 20°$, $\eta_{ma} = 0.85$. $D_p = 0.35$ m.

Net head = Static lift + Friction loss = 40.0 + 2.0 + 6.0 = 48.0 m

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1200}{60} = 31.42 \text{ m/s}$$

By assuming radial flow at inlet, manometric efficiency $\eta_{ma} = \dfrac{g H_m}{u_2 V_{u2}}$

$$0.85 = \frac{9.81 \times 48.0}{31.42 \times V_{u2}}$$

$$V_{u2} = 17.63 \text{ m/s}$$

Centrifugal pumps

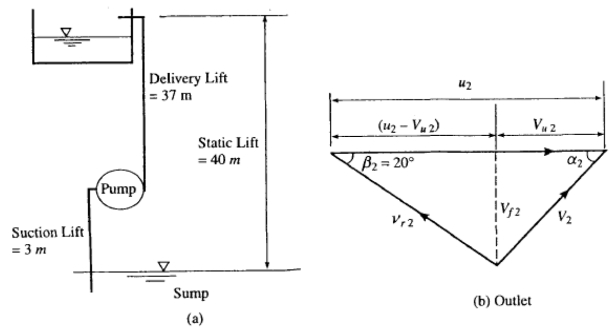

Fig. 5.23 *(a) Schematic layout of the pump; (b) Outlet velocity triangle, Example 5.7*

From outlet velocity triangle, Fig. 5.23(b), $\tan\beta_2 = \dfrac{V_{f2}}{(u_2 - V_{u2})}$

$$\tan 20° = \frac{V_{f2}}{(31.42 - 17.63)} = 0.3639$$

$V_{f2} = 5.02$ m/s

Centrifugal pumps

Discharge $Q = \pi D_2 b_2 V_{f2} = \pi \times 0.5 \times 0.03 \times 5.02 = 0.2366$ m³/s

Velocity in delivery pipe $= V_d =$ Velocity of suction pipe $= V_s$

$$V_s = \frac{Q}{\frac{\pi}{4} D_p^2} = \frac{0.2366}{\frac{\pi}{4} \times (0.35)^2} = 2.459 \text{ m/s}$$

$$\frac{V_s^2}{2g} = \frac{V_d^2}{2g} = \frac{(2.459)^2}{2 \times 9.81} = 0.308 \text{ m}$$

Delivery side of pump: Let the pressure on delivery side $= p_d$

$$\frac{p_d}{\gamma} + \frac{V_d^2}{2g} = h_d + h_{fd} + \frac{V_d^2}{2g}$$

$$\frac{p_d}{\gamma} = h_d + h_{fd} = 37 + 6 = 43.0 \text{ m}$$

$p_d = 43.0 \times 9.79 = 421$ kPa (gauge)

Suction side of pump:

Let the pressure on suction side $= p_s$ and atmospheric pressure $= p_{atm}$. Then

$$\frac{p_{atm}}{\gamma} = h_s + h_{fs} + \frac{p_s}{\gamma} + \frac{V_s^2}{2g}$$

Centrifugal pumps

Taking atmospheric pressure as datum pressure, $0 = 3 + 2 + \frac{p_s}{\gamma} + 0.308$

$$\frac{p_s}{\gamma} = -5.308 \text{ m (vacuum pressure)}$$

$$= -(5.308 \times 9.79) = -51.97 \text{ kPa (vacuum)}$$

Centrifugal pumps

**EXAMPLE 5.8

A centrifugal pump while running at 1000 rpm is required to discharge 65 L/s of water against a total head of 16 m. The manometric efficiency of the pump is 0.85. If the vane angle at the outlet is 35° and the velocity of flow is 1.5 m/s, estimate the outer diameter of the impeller and its width at the exit.

Solution

Given: N = 1000 rpm, Q = 0.065 m^3/s, η_{ma} = 0.85, β_2 = 35°, $V_{f1} = V_{f2}$ = 1.5 m/s, H_m = 16.0 m

Manometric efficiency $\eta_{ma} = \frac{gH_m}{u_2 V_{u2}}$,

$$0.85 = \frac{9.81 \times 16}{u_2 V_{u2}}$$

$$u_2 V_{u2} = 184.66 \qquad \text{(i)}$$

From the outlet velocity triangle,

$$\tan\beta_2 = \frac{V_{f2}}{u_2 - V_{u2}} = \tan 35° = 0.700$$

Centrifugal pumps

$$u_2 - V_{u2} = \frac{1.50}{0.70} = 2.142$$

$V_{u2} = (u_2 - 2.142)$

Substituting in Eq. (i), $u_2^2 - 2.142\, u_2 - 184.66 = 0$

Taking the positive root $u_2 = 14.702$ m/s

$$V_{u2} = \frac{184.66}{14.702} = 12.56 \text{ m/s}$$

Since $u_2 = \frac{\pi D_2 N}{60}$, Impeller diameter $D_2 = \frac{60 \times u_2}{\pi N} = \frac{60 \times 14.702}{\pi \times 1000} = 0.280$ m

Discharge $Q = \pi D_2 b_2 V_{f2}$

$0.065 = \pi \times 0.280 \times b_2 \times 1.5$

b_2 = Width of the impeller at exit = 0.0493 m = 4.93 cm

Centrifugal pumps

***EXAMPLE 5.12

A centrifugal pump has an impeller of 0.5 m outer diameter and when running at 600 rpm discharges 9000 Lpm against a head of 11.0 m. The water enters the impeller radially without whirl or shock. The inner diameter is 0.15 m. The vanes are set back at an angle of 28° to the tangent at the periphery of the outlet. The area of flow is constant from inlet to outlet of the impeller and is 0.05 m². Determine the (a) vane angle at inlet, (b) manometric efficiency of the pump, and (c) minimum speed at which the pump commences to work.

Solution

Given: $Q = \frac{9000}{1000 \times 60} = 0.150$ m³/s, $H_m = 11.0$ m, $N = 600$ rpm, $D_2 = 0.50$ m,

$D_1 = 0.15$ m, $\beta_2 = 28°$, area of flow = 0.05 m²

Figure 5.25 shows the velocity triangles at the inlet and out of the pump.

$$V_{f1} = V_{f2} = \frac{Q}{\text{area}} = \frac{0.15}{0.05} = 3.0 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 600}{60} = 15.71 \text{ m/s}$$

Centrifugal pumps

and $u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.15 \times 600}{60} = 4.71$ m/s

(a) From the inlet velocity triangle, Fig. 5.25, $\tan\beta_1 = \frac{V_{f1}}{u_1} = \frac{3.0}{4.71} = 0.6366$

Vane angle at inlet $\beta_1 = 32.48^o$

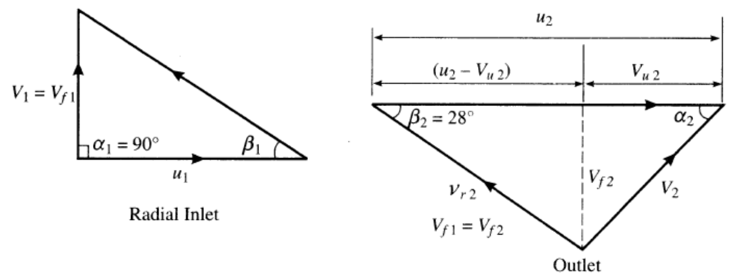

Fig. 5.25 *Velocity triangles, Example 5.12*

Centrifugal pumps

(b) From outlet velocity triangle, Fig. 5.25: $\tan\beta_2 = \frac{V_{f2}}{(u_2 - V_{u2})} = \tan 28° = 0.5317$

$(15.71 - V_{u2}) = 3.0/0.5317 = 5.642$

$V_{u2} = 15.71 - 5.642 = 10.068$ m/s

Manometric efficiency $\eta_{ma} = \frac{gH_m}{u_2 V_{u2}} = \frac{9.81 \times 11}{15.71 \times 10.068} = 0.682$